Escape velocity

Escape velocity is defined as the velocity with which a body has to be projected vertically upwards from the earth's surface so that it escapes the earth's gravitational field altogether.

If v is the escape velocity. then the initial Kinetic energy of projection $\frac{1}{2}mv^2$ must be equal to the work done in moving the body from the surface of the earth to infinity.

Mass of the earth $= M$

Radius of the earth $= R$

Constant of Gravitation $= G$

Mass of the body $= m$

Suppose a body is at a distance x from the earth's centre. The force of gravity acting on the body $= \frac{GMm}{x^2}$

If the body is to be moved through a small distance dx away from the earth's surface work done $= W = \left(\frac{GMm}{x^2}\right)dx$.

work done in moving the body from the surface of the earth to infinity $\int dw = \int_R^\infty \frac{GMm}{x^2} \cdot dx$.

or. $W = \left[\frac{GMm}{R}\right]$

K.E. $= \frac{1}{2}mv^2$, $\therefore \frac{1}{2}mv^2 = \frac{GMm}{R}$, $\therefore v = \sqrt{\frac{2GM}{R}}$

also at the surface of the earth $mg = \frac{GMm}{R^2}$, or $GM = gR^2$

or, $v = \sqrt{2gR}$ The Value of g at the earth's surface $= 9.8 m/s$

$\therefore R = 64 \times 10^5 m$, $\therefore v = \sqrt{2 \times 9.8 \times 64 \times 10^5}$, $\therefore v = 11.2 Km/s$

If a body is projected vertically upwards with a velocity of 11.2 km/s or more it will not return to the earth's surface.