Q14 A particle of mass 'm' is moving under potential $V(x) = ax^3 - bx$, frequency of oscillation about the stable equilibrium position is -

(a) $\left[\frac{12ab}{m^2}\right]^{1/4}$ (b) $\left[\frac{6ab}{m^2}\right]^{1/4}$ (c) $\left[\frac{9ab}{m^2}\right]^{1/4}$ (d) $\left[\frac{3ab}{m^2}\right]^{1/4}$

Sol?

$$V(x) = ax^3 - bx \Rightarrow \left.\frac{dV}{dx}\right|_{x=x_0} = 3ax_0^2 - b$$

$$3ax_0^2 - b = 0 \Rightarrow 3ax_0^2 = b \Rightarrow x_0^2 = \frac{b}{3a} \Rightarrow \boxed{x_0 = \pm\sqrt{\frac{b}{3a}}}$$

So $\pm\sqrt{\frac{b}{3a}}$ are two equilibrium point.

$$\left.\frac{d^2V}{dx^2}\right|_{x_0=\sqrt{\frac{b}{3a}}} = 6ax_0 = 6a\sqrt{\frac{b}{3a}} > 0$$

So $x_0 = \sqrt{\frac{b}{3a}}$ is minima or stable point.

$$\left.\frac{d^2V}{dx^2}\right|_{x_0=-\sqrt{\frac{b}{3a}}} = 6ax_0 = -6a\sqrt{\frac{b}{3a}} < 0$$

So $x_0 = -\sqrt{\frac{b}{3a}}$ is maxima or unstable point.

$\because$ $k = \left.\frac{d^2V}{dx^2}\right|_{x=x_0}$ where x_0 is stable point

So $k = 6a\sqrt{\frac{b}{3a}} = \sqrt{\frac{36a^2b}{3a}} = \sqrt{\frac{36a^2b}{3a}}$

$$= \sqrt{\frac{36ab}{3}}$$

$\therefore$ $\omega = \sqrt{\frac{k}{m}} = \sqrt{\left(\frac{36^2ab}{3m^2}\right)^{1/2}}$

$$\omega = \sqrt{\left(\frac{12ab}{m^2}\right)^{1/2}} = \left(\frac{12ab}{m^2}\right)^{2/4}$$

$$\boxed{\omega = \left(\frac{12ab}{m^2}\right)^{1/4}}$$ Ans

15 In the previous question force on the particle is maximum at -

(a) $x = 0$ (b) $x = \sqrt{\frac{b}{a}}$ (c) $x = \sqrt{\frac{b}{3a}}$ (d) $x = -\sqrt{\frac{b}{3a}}$

Sol^n

$$V(x) = ax^3 - bx$$

$$\vec{F} = -\frac{dV}{dx} = -(3ax^2 - b) = b - 3ax^2$$

for $\vec{F}$ to be max^m or min^m

$$\frac{dF}{dx} = 0$$

$$-6ax = 0$$

$\because -6a \neq 0$

$\therefore \boxed{x = 0}$

$$\frac{d^2F}{dx^2} = -6a \quad (min^m)$$

$\therefore F_{max} = b - 0 = b$ at $\boxed{x = 0}$

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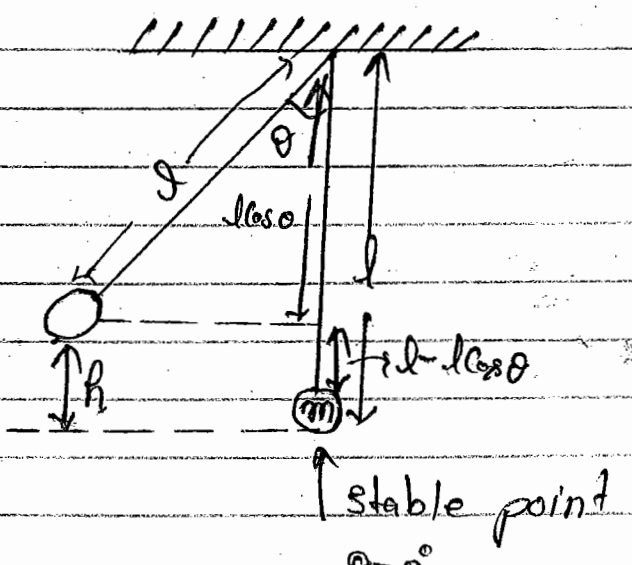
* Time Period of Simple Pendulum :-



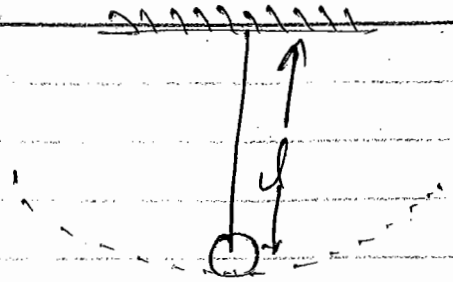
$$P.E = mgh$$

$$\boxed{P.E = mgl(1 - \cos\theta)}$$

$$k = \left.\frac{d^2V}{d\theta^2}\right|_{\theta=\theta_0}$$

$$= \left. mgl\cos\theta \right|_{\theta=0}$$

$$\boxed{K = mgl}$$



$$I = \sum_i m_i r_i^2 = ml^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{mgl}{ml^2}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

Note:-

⇒ K in Linear Case:-

$$K = \left.\frac{d^2V}{dx^2}\right|_{x=0} \qquad \omega = \sqrt{\frac{K}{m}}$$

⇒ K in Angular Case:-

$$K = \left.\frac{d^2U}{d\theta^2}\right|_{\theta=\theta_0} \qquad \omega = \sqrt{\frac{K}{I}}$$