Simple Hormonic motion

- Periodic motion :- repeats itself after regular interval of time. eg:- U.C.M., orbital motion etc
- Ossilatory motion (Harmonic motion) :- To & Fro about fixed point after regular interval of time. eg:- pendulum, spring system
  - All ossilatory motion [with no energy lost] are periodic motion.
  - Condition for any Harmonic motion.

Always towards mean position ← Restoring force

$F = -kx^n$ → (any odd integer i.e. 1, 3, 5...)

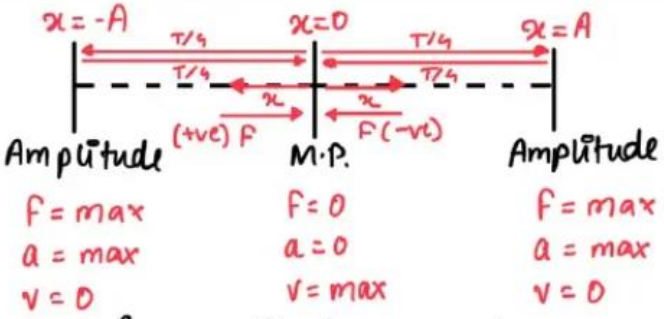
constant

x → displacement from mean position

- when 'n' = 1 → Simple Hormonic motion → straight line

★ S.H.M.

$F = -kx$



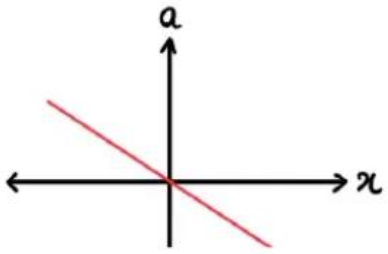
- Amplitude (A) → maximum displacement from mean position
- Time period (T) → Time to complete 1 ossilation
- frequency (F) → Number of ossilations per second
- Angular frequency (ω) → radian / sec.

$$T = \frac{1}{F}, \quad \omega = \frac{2\pi}{T} = 2\pi F$$

$f = -kx$ → for simple Hormonic motion.

- $k = m\omega^2$ $\therefore f = -m\omega^2 x$

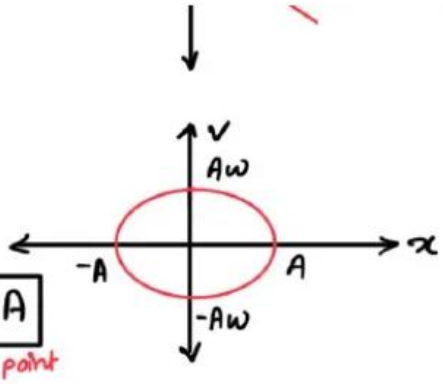
$$a = \frac{f}{m} = \frac{-m\omega^2 x}{m} \quad \therefore a = -\omega^2 x$$



$\therefore$ at extreme $\boxed{a_{max} = -\omega^2 A}$ ← at extreme

$\ominus\ v\frac{dv}{dx} = -\omega^2 x$ $\boxed{\therefore\ v = \omega\sqrt{A^2 - x^2}}$

$\rightarrow \frac{v^2}{(A\omega)^2} + \frac{x^2}{A^2} = 1$ (ellips), $\boxed{v_{max} = \omega A}$ ← at mean point



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* Differential Equation of S·H·M (when displacement is in y-axis)

$a = -\omega^2 x = \frac{d^2x}{dt^2}$ $\boxed{\therefore\ \frac{d^2x}{dt^2} + \omega^2 x = 0}$

* When a motion satisfies this equation it is Simple Harmonic motion.

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* Equation of S·H·M

$\ominus\ x = A\sin(\omega t + \phi)$ — A: Amplitude, t: time, ω: angular frequency, ϕ: Initial phase

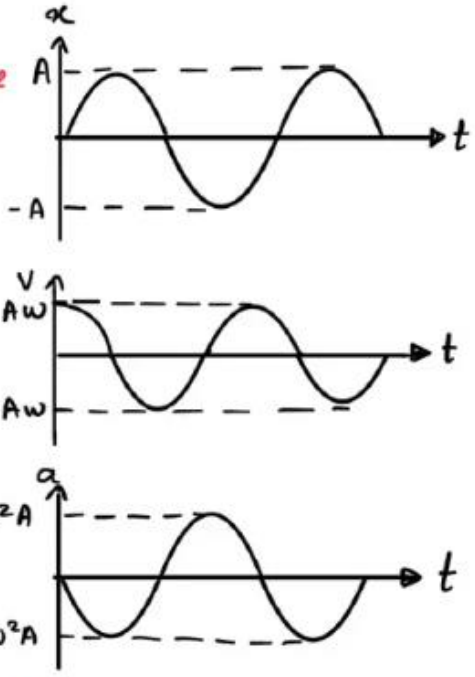
displacement from mean position

$\phi = \sin^{-1}\left(\frac{x}{A}\right)$

$\ominus\ v = A\omega\cos(\omega t + \phi)$

velocity / acceleration

$\ominus\ a = -A\omega^2\sin(\omega t + \phi)$



* Find ϕ using x & v or V-C-M when necessary
* All graphs are general i.e. when $\sin\phi = 0$ & particle starts from origin & move towards right
* Graphs will change as per the value of ϕ and direction of the particle