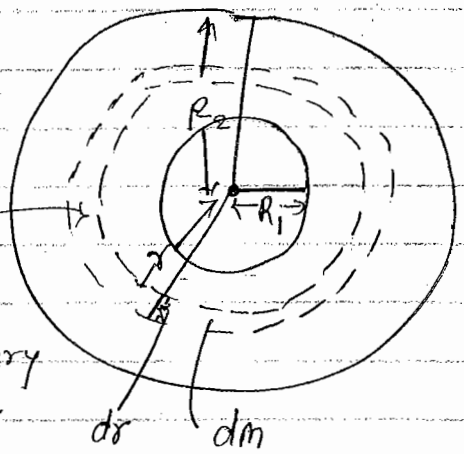
* Moment of Inertia of a big object is to be calculated, we can first calculate M.I. of a small elementary object and integrate it.

Ex-

$$I = \int dI$$

$$dI = \frac{2}{3} dm r^2$$



$$\frac{dm}{dV} = \rho = \frac{M}{\frac{4}{3}\pi(R_2^3 - R_1^3)}$$

$$dm = \frac{3M \cdot dV}{4\pi(R_2^3 - R_1^3)}$$

$$\boxed{dm = \frac{3M}{4\pi(R_2^3 - R_1^3)} dV}$$

$$dV = 4\pi r^2 dr$$

$$dm = \frac{3M}{4\pi(R_2^3 - R_1^3)} 4\pi r^2 dr$$

$$= \frac{3M r^2 dr}{(R_2^3 - R_1^3)}$$

$$= \frac{2}{3} M \int_{R_1}^{R_2} \frac{3r^2 dr}{(R_2^3 - R_1^3)} r^2 = \frac{M}{(R_2^3 - R_1^3)} \times \frac{2}{3} \times 3 \left[\frac{r^5}{5}\right]_{R_1}^{R_2}$$

$$= \frac{2M}{5(R_2^3 - R_1^3)} [(R_2^5 - R_1^5)]$$

$$= \frac{2}{5} M \left(\frac{(R_2^5 - R_1^5)}{(R_2^3 - R_1^3)} \right)$$

Ans

* Dynamics of rigid body :- A rigid body can have following types of motion.

1 Pure Rotation :-

At least one point of object remains at rest.

In this case -

$$K.E \neq \frac{1}{2} m v^2$$

Here, $\boxed{K.E = \frac{1}{2} I \omega^2}$

2. ~~Pure~~ Translation :- ~~It means there is no rotation~~ i.e. $\boxed{\omega = 0}$

$L_{axis} = I_{axis}\, \omega$

$\boxed{\tau = I_{axis}\, \alpha}$ eqn of motion.

$\boxed{\frac{dL_{axis}}{dt} = J_{axis}}$

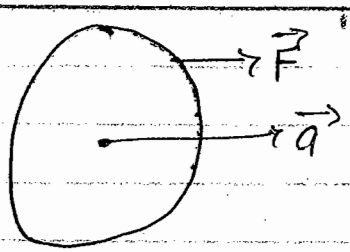
2. Pure Translation :-

It means there is no rotation i.e. $\omega = 0$

$$\boxed{K.E. = \frac{1}{2} m v^2}$$

Equation of motion :-

$$F_{net\ force} = ma$$



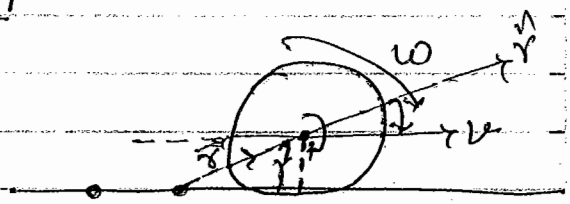
$$L = pr_\perp$$

3. General Motion :-

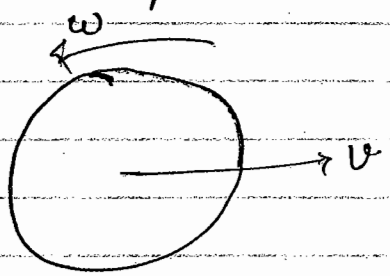
Translation + Rotation

$$K.E. = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

$$L = I\omega \pm pr_\perp$$



$r_\perp = R$ [If L is calculated about a point on path of round object]

$$L = I\omega + pr_\perp$$ { ∵ direction of ω and r x v is same.

$$L = I\omega - pr_\perp$$ { ∵ direction of ω and r x v is opposite }